## PMC RELAY RACES 4/11~ RULES/GUIDELINES

Work with your teammates to solve the problems in each round. Write your answers on the BACK of each round's slip, and call one of the lecturers over when you're ready for us to check. Do not call us over until you're absolutely sure about your answers! If you get any of the questions wrong when we check, we'll let you know which one's wrong, and you can have the chance to figure out another answer. The first team to get all three questions of the round correct will get a point. The winning team will be the one with the most points at the end of all 10 rounds!

## PMC RELAY RACES ~ ROUND 1

1. What digit (numbers $0-9$ ) is the most frequent between the numbers 1 and 1,000 (inclusive)? What about the least frequent?
2. Parth has 14 fruit trees. He has 1 apricot tree and 1 pear tree, and the rest are divided evenly among apple, peach, and plum trees. Last Thursday, Parth picked 6 apples from each apple tree, 10 plums from each plum tree, 3 peaches from each peach tree, and 4 pears from the pear tree. How many pieces of fruit did he pick last Thursday?
3. How many times do the hands of a clock overlap in 24 hours?

## PMC RELAY RACES ~ ROUND 2

4. Arrange the numbers 1 through 9 on a tic tac toe board such that the numbers in each row, column, and diagonal add up to 15 . You may only use each number once.
5. "The day before yesterday I was only 25 and next year I will turn 28. " What day is my birthday?
6. An insurance salesman walk up to house and knocks on the door. A woman answers, and he asks her how many children she has and how old they are. She says I will give you a hint. If you multiply the 3 children's ages, you get 36 . He says this is not enough information. So she gives a him 2nd hint. If you add up the children's ages, the sum is the number on the house next door. He goes next door and looks at the house number and says this is still not enough information. So she says she'll give him one last hint which is that her oldest of the 3 plays piano. How old are each of the woman's children (you should have three numbers written down)?

## PMC RELAY RACES ~ ROUND 3

7. Tom asked his Granny how old she was. Rather than giving him a straight answer, she replied: "I have 6 children, and there are 4 years between each one and the next. I had my first child (your Uncle Peter) when I was 19. Now the youngest one (Your Auntie Jane) is 19 herself. That's all I'm telling you!" How old is Tom's Granny?
8. The box on the right is a Magic Square. This means that the numbers add up to the same total in every direction. Every row, column and diagonal adds up to 111. But there are some numbers missing! Fill in the missing numbers, but keep in mind that they are all different!
9. Abby lists 4 consecutive multiples of some number. The average of the first two multiples is 28 and the average of the last two is 44 . What is the greatest multiple on Abby's list?


## PMC RELAY RACES ~ ROUND 4

10. Find a 10-digit number where the first digit is how many zeros in the number, the second digit is how many 1 is in the number etc. until the tenth digit which is how many 9 s in the number.
11. How can you add eight 8's to get the number 1,000? (HINT: you will need to combine some 8's together to make larger numbers like 8,888)
12. Anjali gives Parth and Jeremy a list of 10 possible dates for her birthday (shown to the right), but she only tells Parth the month of her birthday and Jeremy the day of her birthday. Here's the conversation that Parth and Jeremy had:

| May 15 | May 16 | May 19 |
| :--- | :--- | :--- |
| June 17 | June 18 |  |
| July 14 | July 16 |  |
| August 14 | August 15 | August 17 |

Parth: I don't know when Anjali's birthday is, but I know that Jeremy does not know either.
Jeremy: At first I didn't know when Anjali's birthday is, but now I know.
Parth: Then I also know when Anjali's birthday is.
So, when is Anjali's birthday?

## PMC RELAY RACES ~ ROUND 5

13. Anusha the athlete is able to jump FOREVER. However, every time that she jumps she gets a bit more tired, and every jump goes $1 / 2$ as far as her prior jump. Now, for her very first jump, she goes $1 / 2$ of a foot. On her second jump, she goes $1 / 4$ of a foot, and so on and so forth. How many jumps does it take for her to travel 1 foot?
14. Srikar the merchant can place 8 large boxes or 10 small boxes into a carton for shipping. In one shipment, he sent a total of 96 boxes. If there are more large boxes than small boxes, how many total cartons did he ship?
15. You have to try to find out which card I am thinking of. Here are some clues (using the picture below):
i) The value of my card is a prime number.
ii) The values of my two neighbours add up to a multiple of 3 .
iii) My card is next to a card which is next to the 2 of hearts.


## PMC RELAY RACES ~ ROUND 6

16. Welcome to Parth's Pet Store! If Parth puts one parrot per cage, he has one bird too many. However, if he puts two parrots in each cage, he has one cage too many. How many cages and parrots does Parth have?
17. King Tut died 120 years after King Eros was born. Their combined ages when they died was 100 years. King Eros died in the year 1040. In what year was King Tut born?
18. There are 100 people standing in a circle, numbered 1-100 (in order). Person \#1 has the magical PMC broccoli, which transports a person to another dimension if it touches their shoulder. Person \#1 taps the broccoli on \#2's shoulder (thereby sending him to another dimension) and passes the broccoli onto the next person (\#3). The magical broccoli continues around the circle over and over again in this manner until only one person is left in the circle. Which number is left in the end?

## PMC RELAY RACES ~ ROUND 7

19. Esha's twin lives at the reverse of Esha's house number (ex: 25 and 52). The difference between their house numbers ends in two. What is Esha's lowest possible house number?
20. A small number of cards has been lost from a complete pack. If I deal among four people, three cards remain. If I deal among three people, two remain and if I deal among five people, two cards remain. How many cards are there?
21. What is the smallest whole number that is equal to seven times the sum of its digits?

## PMC RELAY RACES ~ ROUND 8

22. Jeremy noticed that the amount he was paying for his lunch was a rearrangement of the digits of the amount of money he had in his pocket, and that the money he had left over after paying for lunch was yet another rearrangement of the same three digits! How much money did Jeremy start with?
23. Granny Adams left half her money to her granddaughter and half that amount to her grandson. She left a sixth to her brother, and the remainder, $\$ 1,000$, to the dogs' home. How much did she leave altogether?
24. One brother says of his younger brother: "Two years ago, I was three times as old as my brother was. In three years' time, I will be twice as old as my brother." How old are they each now?

## PMC RELAY RACES ~ ROUND 9

25. In my family each child has at least two brothers and at least one sister. What is the smallest possible number of children in my family?
26. Twelve girls met in a coffee shop. On average, they ate 1.5 cupcakes each. None of them ate more than two cupcakes and two of them had only mineral water. How many girls ate two cupcakes?
27. The sum of the digits of a seven-digit number is 6 . What is the product of these digits?

## PMC RELAY RACES ~ ROUND 10

28. In the figure to the right, what is the minimum number of dots that need to be removed from the given figure so that no three of the remaining dots are collinear (on the same line)?
29. While in Verona, Anusha decided to walk each of the five famous bridges on the Adige River. She began and ended her walk in the same place, passed through each of the 5 bridges at least once, and did not use any other bridge. What is the least number of times Anusha have crossed the Adige River?
30. How many three digit numbers have the property that the middle digit is the arithmetic mean of the other two digits?

## PMC RELAY RACES ~ CHALLENGE ROUND

31. How many perfect cubes lie between $2^{8}+1$ and $2^{18}+1$, inclusive?
32. Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?
33. Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips?
